Andrzej Schinzel (1937–2021)

Jerzy Kaczorowski

26 stycznia 2022 r.

Plan wykładu

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- CZĘŚĆ I : Fakty z życia prof. Andrzeja Schinzla.
- CZĘŚĆ II: O matematyce prof. Andrzeja Schinzla ('look through a keyhole').
- CZĘŚĆ III: O Andrzeju Schinzlu prywatnie.

Fakty z życia prof. Andrzeja Schinzla.

- $\star$ 5 kwietnia 1937, Sandomierz.
- † 21 sierpnia 2021, Konstancin-Jeziorna.
- Rodzice: Zygmunt (1887–1974), lekarz; Wanda Maria z d. Świeżyńska (1905–2001), artystka malarka.
- Miejsce wiecznego spoczynku: Cmentarz Katedralny w Sandomierzu.

Fakty z życia prof. Andrzeja Schinzla.

- Laureat II Olimpiady Matematycznej (1950/51 - pierwsze miejsce).
- Studia: 1958 Wydział Matematyczny Uniwersytetu Warszawskiego.
- 1960 doktorat pod kierownictwem prof. Wacława Sierpińskiego *O pewnych zagadnieniach z arytmetycznej teorii ułamków łańcuchowych*, IMPAN.
- 1960–1961 stypendium Fundacji Rockefellera w Cambridge i Uppsali.
- habilitacja 1962, *O dzielnikach pierwszych liczb Lehmera*, IMPAN.
- profesura nadzwyczajna 1967,
- profesura zwyczajna 1974.

Fakty z życia prof. Andrzeja Schinzla.

- Laureat II Olimpiady Matematycznej (1950/51 - pierwsze miejsce). [14 l.]
- Studia: 1958 Wydział Matematyczny Uniwersytetu Warszawskiego. [21 l.]
- 1960 doktorat pod kierownictwem prof. Wacława Sierpińskiego *O pewnych zagadnieniach z arytmetycznej teorii ułamków łańcuchowych*, IMPAN. [23 l.]
- 1960–1961 stypendium Fundacji Rockefellera w Cambridge i Uppsali.
- habilitacja 1962, *O dzielnikach pierwszych liczb Lehmera*, IMPAN.[25 l.]
- profesura nadzwyczajna 1967, [30 l.]
- profesura zwyczajna 1974. [37 l.]

Fakty z życia prof. Andrzeja Schinzla.

- członek korespondent PAN (1979),
- członek rzeczywisty PAN (1994),
- członek czynny Polskiej Akademii Umiejętności (2004),
- członek honorowy Węgierskiej Akademii Nauk(2001),
- członek korespondent Austriackiej Akademii Nauk (1997),
- członek Niemieckiej Akademii Przyrodników Leopoldina (1976).
- członek zwyczajny Towarzystwa Naukowego Warszawskiego,
- członek honorowy Polskiego Towarzystwa Matematycznego.

Fakty z życia prof. Andrzeja Schinzla.

Od 1960 do 2019 roku zatrudniony był w Instytucie Matematycznym PAN w Warszawie. Wywarł ogromny wpływ na naukowy profil Instytutu, prowadząc intensywną pracę badawczą, ale także pełniąc odpowiedzialne funkcje kierownicze. Od 1968 roku był kierownikiem Działu Teorii Liczb, w latach 1986-1989 zastępcą dyrektora do spraw naukowych, w latach 2007-2018 przewodniczącym Rady Naukowej, a od 2019 roku aż do śmierci – jej honorowym przewodniczącym.

Fakty z życia prof. Andrzeja Schinzla.

Był wielokrotnie zapraszany do wygłaszania wykładów na wielu zagranicznych uniwersytetach oraz na prestiżowych konferencjach naukowych (na przykład Journées Arithmétiques, Bordeaux 1974, Caen 1976, Exeter 1980, Ulm 1987, Limoges 1997). Został również zaproszony do wygłoszenia wykładu na Międzynarodowym Kongresie Matematyków w Nicei w 1970 r. Otrzymał Nagrodę Wydziału Nauk Matematyczno-Fizycznych, Chemicznych i Geologiczno-Geograficznych PAN (1962), Nagrodę Polskiego Towarzystwa Matematycznego im. Stanisława Zaremby (1969), Nagrodę Państwową II stopnia (1974), Nagrodę Fundacji Alfreda Jurzykowskiego (Nowy Jork, 1979), Medal Komisji Edukacji Narodowej (1973), Medal im. Stefana Banacha PAN (1992), Medal im. Władysława Orlicza UAM (1997), Nagrodę Prezesa Rady Ministrów za wybitny dorobek naukowy w 2003 roku, oraz Papieski Krzyż Pro Ecclesia et Pontifice (1977).

Fakty z życia prof. Andrzeja Schinzla.

Doktoraty honorowe:

- 1998 Uniwersytet Caen,
- 2012 Uniwersytet im. Adama Mickiewicza w Poznaniu
- 2012 Uniwersytet Kardynała Stefana Wyszyńskiego w Warszawie.

Fakty z życia prof. Andrzeja Schinzla.

Inne aktywności:

- członek Komitetu Nagród Fieldsa (1979–82) Międzynarodowej Unii Matematycznej,
- członek komitetów naukowych kongresów Europejskiego Towarzystwa Matematycznego w Budapeszcie (1996) i Sztokholmie (2004),
- członek Komitetu Głównego Olimpiady Matematycznej (przewodniczący w latach 1969-1971 i 1995-1997),
- wiceprezes PTM (1981-1983), od 2009 – członek honorowy,
- jeden z twórców Towarzystwa Naukowego Sandomierza. W 2016 roku otrzymał honorowe obywatelstwo tego miasta.

Fakty z życia prof. Andrzeja Schinzla.

- Członek i protektor czynny Archikonfraterni Literackiej w Warszawie. Trzykrotnie (1993, 2001, 2003) uczestniczył w letnich seminariach w rezydencji papieskiej w Castel Gandolfo na zaproszenie Jana Pawła II.
- Odznaczenia: Krzyż Kawalerski OOP (1972), Krzyż Oficerski OOP (1988), Krzyż Komandorski OOP (2002).

Basic facts about Andrzej Schinzel's work in number theory

- Earliest Indexed Publication: 1954
(Andrzej Schinzel was 17 at that time)
- Total Publications: 359
(the first thirty were published while he was an undergraduate student)
- Total Citations (2022):
1897 (MathSciNet)
- Erdős number: 1

Basic facts about Andrzej Schinzel's work in number theory

- The main subject inside number theory: polynomials
  'Zajmowałem się wieloma tematami. Moje najlepsze prace dotyczą wielomianów.' (Interview with A.S. in Academia nr 2 (30) 2012)
- Other subjects:
  Algebraic geometry, Combinatorics, Difference and functional equations, Field theory, Functions of a complex variable, History and biography, Information and communication, Linear and multilinear algebra, matrix theory

Basic facts about Andrzej Schinzel's work in number theory

Coauthors (87)

Aliev, Iskander M.; Baker, Alan; Baron, Gerd; Barsky, Daniel; Bergman, Stefan; Bhaskaran, M.; Białynicki-Birula, Andrzej S.; Birch, Bryan J.; Bolis, Theodore S.; Brostow, Witold; Browkin, Jerzy (8); Bézivin, Jean-Paul; Cassels, J. W. S.; Chaładus, Stefan; Choudhry, Ajai; Chowla, Sarvadaman; Davenport, Harold (8); Diviš, Bohuslav; Dobrowolski, Edward; Duda, Roman; Erdős, Paul; Filaseta, Michael; Fried, Michael David; Goldfeld, Dorian M.; Gołąb, Stanisław; Granville, Andrew James; Greaves, George; Grużewski, Aleksander; Grytczuk, Aleksander; Győry, Kálmán; Hajdu, Lajos; Hall, Marshall, Jr.; Hanél, Jaroslav; Hartman, Stanisław; Jakubec, Stanislav; Kanemitsu, Shigeru; Kiss, Péter; Knaster, Bronisław; Lawton, Wayne M.; Lewis, Donald John; Losonczi, László; Mikusiński, Jan G.; Misiurewicz, Michał;

Basic facts about Andrzej Schinzel's work in number theory

Coauthors (87), continuation

Montgomery, Hugh L.; Mycielski, Jan; Mąkowski, Andrzej; Narkiewicz, Władysław; Nicolas, Jean-Louis ; Niederreiter, Harald; Oderfeld, Jan; Paszkiewicz, Andrzej; Patéka, Milan; Perlis, Robert; Pintér, Ákos; Pomerance, Carl; Postnikova, L. P.; Péter, Gyöngyvér; Rokowska, Barbara; Rolewicz, Stefan; Rotkiewicz, Andrzej; Rubel, Lee A.; Ruzsa, Imre Z.; Schlickewei, Hans Peter; Schmidt, Wolfgang M. (6); Shorey, Tarlok N.; Sierpiński, Wacław (9); Skałba, Mariusz; Somer, Lawrence; Spież, Stanisław; Szekeres, George; Tanigawa, Yoshio; Tijdeman, Robert; Tverberg, Helge A.; Ulas, Maciej; Urbanowicz, Jerzy; Wakulicz, Andrzej; Wang, Yuan; Wirsing, Eduard A.; Wróblewski, Jarosław; Wójcik, Jan; Zakarczemny, Maciej; Zannier, Umberto M.; Zassenhaus, Hans J.; van Wamelen, Paul B.; van der Poorten, Alfred Jacobus; Šalát, Tibor; Šustek, Jan.

Basic facts about Andrzej Schinzel's work in number theory

PhD students

1. Jan Wójcik, Polska Akademia Nauk 1967
2. Henryk Iwaniec, Uniwersytet Warszawski 1972
3. Rolf Wasen, IMPAN 1977
4. Adam Bazylewicz, IMPAN 1978
5. Jacek Fabrykowski, IMPAN 1980
6. Iskander Aliev, IMPAN 2001
7. Roberto Avanzi, UniversitĂt Duisburg-Essen 2001
8. Adam Grygiel, IMPAN 2010
9. Maciej Zakarczemny, IMPAN 2012

Basic facts about Andrzej Schinzel's work in number theory

(Almost) complete description of A. S.'s work

1. W. Narkiewicz, *The work of Andrzej Schinzel in number theory,* Number theory in progress, Vol. 1 (Zakopane–Kościelisko, 1997), 341—357, de Gruyter, Berlin, 1999.

2. Andrzej Schinzel *Selecta*, Heritage of European Mathematics. European Mathematical Society (EMS), Zürich, 2007. 1393 pages, edited by Henryk Iwaniec, Władysław Narkiewicz and Jerzy Urbanowicz.

   Vol. I. Diophantine problems and polynomials.
   Vol. II. Elementary, analytic and geometric number theory.

Basic facts about Andrzej Schinzel's work in number theory

R. C. Baker (Math. Rev.): *'The breadth and depth of Schinzel's work are both remarkable.'*

About the title of this section

It's impossible to give a detailed account of all the scientific accomplishments of Andrzej Schinzel in a short talk.
Therefore: 'glance through a keyhole'

You can't see much that way!

The first look through the keyhole

First published papers (A.S. is 17 years old)

- Schinzel, A., *Sur la décomposition des nombres naturels en sommes de nombres triangulaires distincts,* Bull. Acad. Polon. Sci. Cl. III. 2 (1954), 409–410.
- Schinzel, A., Sierpiński, W., *Sur quelques propriétés des fonctions* $\varphi(n)$ *et* $\sigma(n)$, Bull. Acad. Polon. Sci. Cl. III. 2 (1954), 463–466.
- Schinzel, A., *Quelques thórèmes sur les fonctions* $\varphi(n)$ *et* $\sigma(n)$, Bull. Acad. Polon. Sci. Cl. III. 2 (1954), 467–469.
- Schinzel, A. *Sur une propriété du nombre de diviseurs*, Publ. Math. Debrecen 3 (1954), 261–262.
- Schinzel, A. *Generalisation of a theorem of B.S.K.R. Somayajulu on the Euler's function* $\varphi(n)$, Ganita 5 (1954), 123–128.

The first look through the keyhole: about paper 3.

Theorem

Given any positive integers m and k, there exist integers n and h such that

$$\frac{\varphi(n+i)}{\varphi(n+i-1)} > m \quad \text{and} \quad \frac{\varphi(h+i-1)}{\varphi(h+i)} > m$$

for $i = 1, 2, \ldots, k$. Analogous results hold for the function $\sigma(n)$.

Next year (1955) Schinzel generalized this result as follows [Bull. Acad. Polon. Sci. Cl. III. 3 (1955), 415–419.]

The first look through the keyhole: about paper 3.

Theorem

Let $a_1, a_2, \ldots, a_h$ be any finite sequence of non-negative numbers or infinity. Then there exists an infinite sequence $n_1 < n_2 < n_3 < \ldots$ of natural numbers such that

$$\lim_{k\to\infty} \frac{\varphi(n_k + i)}{\varphi(n_k + i - 1)} = a_i$$

for $i = 1, 2, \ldots, h$. The same result holds for $\sigma(n)$.

The first look through the keyhole: about paper 3.

This theorem was then generalized by many authors including Paul Erdős. After more than 50 years it still inspires mathematicians, cf. for instance [Alkan, Emre; Ford, Kevin; Zaharescu, Alexandru, *Diophantine approximation with arithmetic functions. I*, Trans. Amer. Math. Soc. 361 (2009), no. 5, 2263–2275], [Bengoechea, Paloma, *Metric number theory of Fourier coefficients of modular forms*, Proc. Amer. Math. Soc. 147 (2019), no. 7, 2835–2845.]

The second look through the keyhole

The most frequently cited paper (103 times/MR)

- Schinzel, A., Sierpiński, W. *Sur certaines hypothèses concernant les nombres premiers*, Acta Arith. 4 (1958), 185–208; erratum 5 (1958), 259.
- "Hypothesis H": *if* $f_1(x), f_2(x), \ldots, f_r(x)$ *are integral-valued polynomials,*

$$\prod_{j}^{r} f_j(a) \neq 0 (\bmod p)$$

for any prime p *and some* a*, then there exist infinitely many integers* n *for which*

$$f_1(n), f_2(n), \ldots, f_r(n)$$

are all primes.

The second look through the keyhole

The most frequently cited paper (103 times/MR)

- Hypothesis H is one of the most challenging open problems in the prime number theory.
- It is the common generalization of a conjecture of Dickson (1904; the case of linear polynomials) and Buniakowski (1857; the case of a single polynomial). Quantitative versions of H were also formulated (Bateman-Horn, 1962).
- The only instance where Hypothesis H is proved is the classical Dirichlet theorem on primes in arithmetic progression;

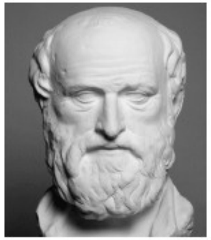
$$f(x) = ax + b, \quad (a, b) = 1, \quad a > b \geqslant 1$$

'There are infinitely many primes in every arithmetic progression $an + b$, $n = 1, 2, 3, \ldots$, *where* a *and* b *are arbitrary fixed coprime positive integers,'* (Dirichlet, 1837).

The second look through the keyhole

The most frequently cited paper (103 times/MR)

- Non-trivial 'approximations' to H can be obtained by sieve methods.
- The roots of sieve methods date back to antiquity (the sieve of Eratosthenes), the modern formulation was developed by Viggo Brun in 1915.



Eratosthenes

Viggo Brun

The second look through the keyhole

The most frequently cited paper (103 times/MR)

A sample 'approximation' to H that can be obtained by sieve methods.

Theorem (G. Ricci, 1937)

If polynomials $f_1(x), \ldots, f_r(x)$ satisfy conditions of Hypothesis H, there exists a positive integer k such that all numbers

$$f_1(n), \ldots, f_r(n)$$

are products of at most k primes for infinitely many positive integers n.

For many similar results see [H. Halberstam, H.-E. Richert, *Sieve Methods*, Academic press, 1974].

The second look through the keyhole

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If polynomials $f_1(x), \ldots, f_r(x)$ satisfy conditions of Hypothesis H, there exists a positive integer k such that all numbers

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For many similar results see [H. Halberstam, H.-E. Richert, *Sieve Methods*, Academic press, 1974].

The second look through the keyhole

The most frequently cited paper (103 times/MR)

- Hypothesis H has many amazing consequences.We quote just two of them.
- The first, which is now a theorem, and the second, which is now 'almost proved', and which is presently a very hot subject in analytic number theory.

The second look through the keyhole

The most frequently cited paper (103 times/MR)

- Sierpiński's hypothesis (50'): *for every $k > 1$ there exists a positive integer m such that the equation $\varphi(x) = m$ has exactly k solutions.*
  True: Kevin Ford, Ann. of Math. (2) 150 (1999), no. 1, 283–311.
- Bounded gaps between primes: *There exists a positive integer k such that there are infinitely many primes p for which $p + 2k$ is a prime.*
  True: Yitang Zhang, Ann. of Math. (2) 179 (2014), no. 3, 1121–1174; $k \leqslant 3,5 \times 10^7$).
  Important contributors: D. Goldston, C. Yıldırım, J.Pintz, J. Motohashi, J. Maynard. T. Tao and Polymath Team.
  Hypothesis H predicts that this happens for all integer $k \geqslant 1$.
  The case $k = 1$ is the famous Twin Prime Conjecture.

The third look through the keyhole

The longest series of papers (12 parts published over 30 years).

Reducibility of lacunary polynomials. I – XII.
I.Acta Arith. 16 1969/1970 123–159,
II. Acta Arith. 16 1969/1970 37–392,
III. Acta Arith. 34 (1977/78), no. 3, 227–266,
IV. Acta Arith. 43 (1984), no. 3, 313–315,
V. Acta Arith. 43 (1984), no. 4, 425–440,
VI. Acta Arith. 47 (1986), no. 3, 277–293,
VII. Monatsh. Math. 102 (1986), no. 4, 309–337,
VIII. Acta Arith. 50 (1988), no. 1, 91–106,
IX. New advances in transcendence theory (Durham, 1986), 313–336, Cambridge Univ. Press, Cambridge, 1988,
X. Acta Arith. 53 (1989), no. 1, 47–97,
XI. Acta Arith. 57 (1991), no. 2, 165–175,
XII. Acta Arith. 90 (1999), no. 3, 273–289.

The third look through the keyhole

The longest series of papers (*Reducibility of lacunary polynomials.*).

Notation:
For a polynomial $F(x_1, \ldots, x_k)$ let $JF(x_1, \ldots, x_k)$ denote F deprived of all its factors of the form x^m;
$KF(x_1, \ldots, x_k)$ $(LF(x_1, \ldots, x_k))$ denote F deprived of all its irreducible (over $\mathbb{Q}$) monic cyclotomic (resp. reciprocal) factors as well as of factors of the form x^m.
Moreover, $||F||$ is the sum of squares of the absolute values of coefficients of F; if $F \neq 0$, $|F|$ is the maximum of the degrees of F with respect to x_i $(i = 1, \ldots, k)$.

$$|F|^* = \sqrt{\max\{|F|^2, 2\} + 2}.$$

The third look through the keyhole

Theorem

For any polynomial $F \neq 0$ and any integer $n \neq 0$ there exist integers ν and u such that

$$0 \leqslant \nu \leqslant \exp(10|F| \log |F|^* \log ||F||)^2,$$

$$n = u\nu,$$

$$KF(x^\nu) = const \prod_{\sigma=1}^{s} F_\sigma(x)^{e_\sigma} \text{ implies } JF(x^n) = const \prod_{\sigma=1}^{s} F_\sigma(x^u)^{e_\sigma}.$$

(Factors F_σ are irreducible, coprime and monic.)

This is a special case of the main result of part 1 saying roughly that there are only finitely many types of factorizations of $KF(x_1^{n_1}, \dots, x_k^{n_k})$ for all integral vectors $[n_1, \dots, n_k] \neq [0, \dots, 0]$ such that $F(x_1^{n_1}, \dots, x_k^{n_k}) \neq 0$.

The third look through the keyhole

Corollary

For any polynomial $f(x) \neq 0$ the number of its irreducible non-reciprocal factors except x counted with their multiplicities does not exceed

$$\exp_{||f||^2-5||f||+7}(||f||+2),$$

(a bound independent of $|f|$).

Notation: $\exp_1(x) = \exp(x)$, $\exp_j(x) = \exp(\exp_{j-1}(x))$

The third look through the keyhole

Further sample results from *Reducibility of lacunary polynomials*.

Theorem

Let $a_0, \dots, a_k$ be given non-zero algebraic numbers, and let

$$\mathsf{K} = \mathbb{Q}(a_1/a_0, \dots, a_k/a_0).$$

Assume moreover that $a_0 \in \mathsf{K}$. Then the number of integer vectors $[n_1, \dots, n_k] \in \mathbb{N}^k$, $0 < n_1 < \dots < n_k \leqslant N$ such that $K(a_0 + \sum_{j=1}^k a_j x^{n_j})$ is reducible over K is less than

$$c_0(a_0, \dots, a_k) N^{\frac{k+1}{2}}.$$

In particular, for $k \geqslant 2$ the polynomial $K(a_0 + \sum_{j=1}^k a_j x^{n_j})$ is irreducible with probability one.

The third look through the keyhole

Theorem

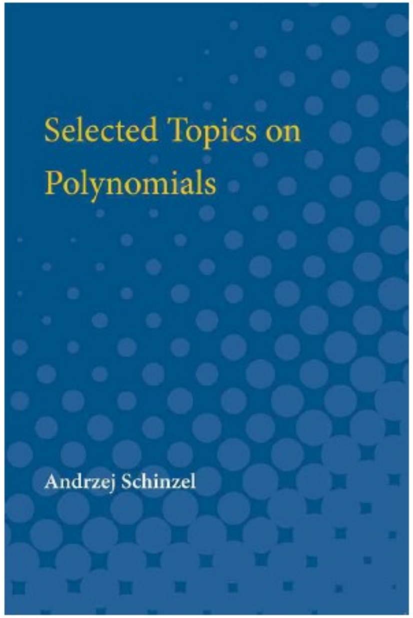
For any non-zero integers A and B, and any polynomial $f(x)$ with integral coefficients, such that $f(0) \neq 0$ and $f(1) \neq -A - B$, there exist infinitely many irreducible polynomials $Ax^m + Bx^n + f(x)$ with $m > n > |f|$. One of them satisfies
$m < \exp((5|f| + 2\log|AB| + 7)(||f|| + A^2 + B^2))$.

Corollary

For any polynomial $f \in \mathbb{Z}[x]$ there exist infinitely many irreducible polynomials $g \in \mathbb{Z}[x]$ such that $||f - g|| \leqslant 2$ if $f(0) \neq 0$, and $||f - g|| \leqslant 3$ always. One of them, g_0, satisfies
$|g_0| < \exp((5|f| + 7)(||f|| + 3))$.

Needless to say papers I–XII contain many interesting results of this or a similar type.

The fourth look through the keyhole: monographs



Selected topics on polynomials, University of Michigan Press, Ann Arbor, Mich., 1982. xxi+250 pp.

The fourth look through the keyhole: monographs

Selected topics on polynomials, University of Michigan Press, Ann Arbor, Mich., 1982. xxi+250 pp.

The book is divided into two parts:

- I. algebraic (polynomials with coefficients in an arbitrary field, sometimes algebraically closed)
- II. arithmetic (coefficients in an algebraic number field).

Complete proofs of both new results and original work on polynomials and Diophantine equations are presented here for the first time in book form.

The fourth look through the keyhole: monographs

Selected topics on polynomials.

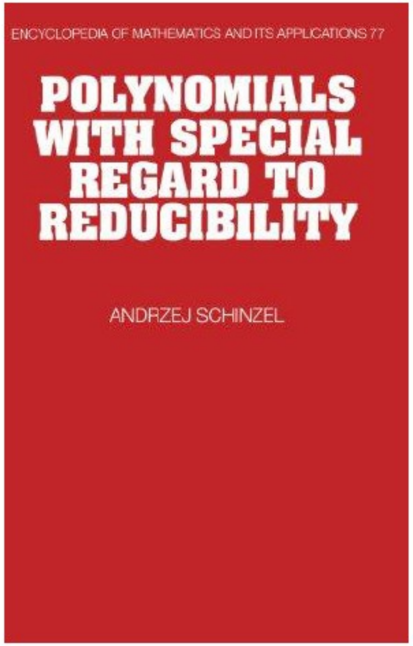
Lists of topics in Part I: (1) Lüroth's theorem and its consequences for polynomials in many variables; (2) Ritt's theory of the composition of polynomials in one variable with application to Diophantine equations; (3) Kronecker's theorem on the connection between coefficients of factors and of the product of polynomials; (4) reducibility of polynomials typified by $F(x, y) + G(z)$; (5) elimination theory for systems of homogeneous equations with applications to the algebra of polynomials; (6) polynomials reducible in an algebraically closed field for every value of some of the variables; (7) equivalence between reducibility of a polynomial in one variable over a field and solvability of a suitable equation in the same field; (8) Capelli's criterion for the reducibility of binomials, its applications and an extension; (9) reducibility of polynomials of the form $F(x_1^{n_1}, \ldots, x_k^{n_k})$.

The fourth look through the keyhole: monographs

Selected topics on polynomials

Lists of topics in Part II:
(10) estimates for the product of zeros of a polynomial outside the unit circle with application to problems of reducibility;
(11) Hilbert's irreducibility theorem and related problems about Diophantine equations with parameters.

The fourth look through the keyhole: monographs



Polynomials with special regard to reducibility. With an appendix by Umberto Zannier, Encyclopedia of Mathematics and its Applications, 77. Cambridge University Press, Cambridge, 2000. x+558 pp.

The fourth look through the keyhole: monographs

Polynomials with special regard to reducibility.

- Contents

  Chapter 1: Arbitrary polynomials over an arbitrary field.
  Chapter 2: Lacunary polynomials over an arbitrary field.
  Chapter 3: Polynomials over an algebraically closed field.
  Chapter 4: Polynomials over a finitely generated field.
  Chapter 5: Polynomials over a number field.
  Chapter 6: Polynomials over a Kroneckerian field.
  + 11 Appendices (one by Umberto Zannier).
- As in *Selected topics on polynomials* many results on the reducibility of polynomials are presented, usually in their most general form. Nearly half of all theorems are due to the author himself.

The fourth look through the keyhole: monographs

Polynomials with special regard to reducibility.

From reviews:

- *'... interesting and original ... contains much material that is unavailable elsewhere.'*
  David W. Boyd, Zentralblatt MATH

- *'This is a wonderful book, filled with unexpected results.'*
  A. von der Poorten, Niew Archief voor Wiskunde

The fifth look through the keyhole

The most 'unusual' paper.

- The subject belongs to general topology.
- Schinzel, A., *A nonstandard metric in the group of reals*, Colloq. Math. 50 (1986), no. 2, 241–248.
  This paper corrects an incorrect statement from
- Hartman, S.; Mycielski, Jan; Rolewicz, S.; Schinzel, A., *Concerning the characterization of linear spaces*, Colloq. Math. 13 1964/1965 199–208.

The fifth look through the keyhole: 'unusual' paper.

Theorem

For $\alpha \in \mathbb{R}$ let

$$N(\alpha) = \inf \left(\sum_{i=1}^{\infty} \frac{1}{\log\log(2a_i)} \right),$$

where the infimum (possibly infinite) is taken over all representations of α in the form $\alpha = \sum_{i=1}^{\infty} \varepsilon_i a_i^{-1}$, $\varepsilon_i = \pm 1$, $a_i > 1$ (integers),and let

$$\rho(x, y) = (1 + N(x - y)^{-1})^{-1},$$

(where $\infty^{-1} = 0$). Then the metric space $(\mathbb{R}, \rho)$ is complete, non separable and nondiscrete. Moreover, the function $\alpha \mapsto \rho(\alpha, 0)$ is a measurable Baire function.

Andrzej Schinzel and ACTA ARITHMETICA

ACTA ARITHMETICA

a S. LUBELSKI et A. WALFISZ fundata, a W. SIERPIŃSKI continuata

Facts that speak for themselves

- Member of the Editorial Board since volume 12(1966–1967).
- Editor for volumes 17(1970) to 130(2007).
- Afterward still in the strict Editorial Board (current volume in print 180(2017)).
- More than 50 years of exemplary and most respectable service to the number theory community.
- 169 volumes of AA published since 1966 (114 as the Editor).
- More than 68,500 printed pages of high quality mathematics.

Prywatnie...

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